Teacher notes
Topic E

How do we calculate energy released: using masses or using binding energies?

Caesium ($^{137}_{55}\text{Cs}$) decays into Barium (Ba) by beta minus decay.

(a) Write down the equation for the decay.
(b) The *atomic* mass of Cs-137 is 136.907089473 u and that of Ba-137 is 136.905827384 u. Show that the total energy released in the decay is about 1.2 MeV.
(c) The binding energy per nucleon of Cs-137 is 8.3890 MeV. Calculate the binding energy per nucleon for Ba-137.
(d) Comment on the result in (c).
(e) Energy released is also calculated by using binding energies. How would you calculate the energy released using binding energies in this decay?
(f) Explain why the kinetic energy of the electron varies from 0 to 1.2 MeV.
(g) What is the energy released in $^{2}_{1}\text{H}+{}^{3}_{1}\text{H}\to{}^{1}_{0}\text{n}+{}^{4}_{2}\text{He}$ using binding energies?

(a) $^{137}_{55}\text{Cs}\to{}^{137}_{56}\text{Ba}+e^{-}+\bar{\nu}$
(b) The standard way to calculate the energy released is to find the total mass on the left, subtract the total mass on the right and multiply by c^2. We have been given atomic masses and we need to use nuclear masses. On the left of the equation we need to subtract 55 electron masses. On the right we need to subtract 56 electron masses for barium and add one electron mass for the electron produced. So in the difference the electron masses cancel out and we can use the given atomic masses. Hence $Q=(136.907089473-136.905827384)\times 931.5=1.18\text{ MeV}$.
(c) $\mu = (ZM_{\text{H}} + Nm_{\text{n}} - M_{\text{atom}}) = 56\times1.007825 + (137 - 56)\times1.008665 - 136.905827384 = 1.2342$ u. The binding energy is $1.2342\times 931.5 = 1.1497\times10^{3}$ MeV and so the binding energy per nucleon is 8.3919 MeV.
(d) We expected the binding energy per nucleon for Ba to be greater than that of caesium. Binding energy per nucleons is a measure of how stable/tightly bound a nucleus is. Caesium becomes more stable by turning itself into a nucleus of barium and barium must be more tightly bound.
(e) The energy released is $(M_{\text{Cs}}-M_{\text{Ba}}-m_e-m_{\bar{\nu}})c^2\approx(M_{\text{Cs}}-M_{\text{Ba}}-m_e)c^2$. We know that

$$BE_{\text{Cs}}=(55m_p+82m_n-M_{\text{Cs}})c^2\Rightarrow M_{\text{Cs}}c^2=(55m_p+82m_n)c^2-BE_{\text{Cs}}$$
$$BE_{\text{Ba}}=(56m_p+81m_n-M_{\text{Ba}})c^2\Rightarrow M_{\text{Ba}}c^2=(56m_p+81m_n)c^2-BE_{\text{Ba}}$$

and so

$$
\begin{aligned}
Q &= (55m_p + 82m_n)c^2 - BE_{\text{Cs}} - (56m_p + 81m_n)c^2 + BE_{\text{Ba}} - m_e c^2 \\
&= BE_{\text{Ba}} - BE_{\text{Cs}} + m_n c^2 - m_p c^2 - m_e c^2 \\
&= BE_{\text{Ba}} - BE_{\text{Cs}} + (m_n - m_p - m_e)c^2
\end{aligned}
$$

We see that the energy released is **not just the difference of binding energies**. This happens when we have weak interaction processes in which electrons and neutrinos appear and the proton number changes.

$$
\begin{aligned}
Q &= 137 \times (8.3919 - 8.3890) + (939.5654 - 938.2721 - 0.5110) \\
&= 0.3973 + 0.7823 \\
&= 1.1796 \approx 1.18 \text{ MeV}
\end{aligned}
$$

just as before.

(f) The released energy is shared by 3 objects: the barium nucleus, the electron and the antineutrino. The barium being very heavy hardly gets any energy. So the energy is mainly shared by the electron and the antineutrino. The share obtained by the electron depends on the direction of velocities and so is not constant.

(g) The energy released is $(M_{\text{H1}} + M_{\text{H2}} - m_n - m_{\text{He4}})c^2$. We know that

$$
\begin{aligned}
BE_{\text{H2}} &= (m_p + m_n - M_{\text{H2}})c^2 \Rightarrow M_{\text{H2}}c^2 = (m_p + m_n)c^2 - BE_{\text{H2}} \\
BE_{\text{H3}} &= (m_p + 2m_n - M_{\text{H2}})c^2 \Rightarrow M_{\text{H3}}c^2 = (m_p + 2m_n)c^2 - BE_{\text{H3}} \\
BE_{\text{He4}} &= (2m_p + 2m_n - M_{\text{He4}})c^2 \Rightarrow M_{\text{He4}}c^2 = (2m_p + 2m_n)c^2 - BE_{\text{He4}}
\end{aligned}
$$

so that

$$
\begin{aligned}
Q &= (M_{\text{H1}} + M_{\text{H2}} - m_n - m_{\text{He4}})c^2 \\
&= (m_p + m_n)c^2 - BE_{\text{H2}} + (m_p + 2m_n)c^2 - BE_{\text{H3}} - m_n c^2 - \left((2m_p + 2m_n)c^2 - BE_{\text{He4}}\right) \\
&= BE_{\text{He4}} - BE_{\text{H2}} - BE_{\text{H3}} \\
&= \text{binding energy on the right} - \text{binding energy on the left}
\end{aligned}
$$